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Mutual information between the main foreign subindices: The application of copula entropy around WHO's declaration date at the time of the COVID-19 pandemic

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ABSTRACT

Objective: The objective of this article is to investigate the dependencies between selected European sub-indices before and during the COVID-19 pandemic.

Research Design & Methods: The main analysis was quantitative. We used copula entropy and Pearson's correlation. We considered the closing prices of sectoral indices from France (CAC sectors), Germany (DAX sectors), the UK (FTSE sectors), and the US (SP sectors), along with the main indices from these countries, that is CAC40, DAX, SP500, and FTSE100 (we collected the data from the database investing.com for the period from 4 January 2017 to 30 March 2023). We performed all analyses using R along with supplementary packages.

Findings: When it comes to indications of the strength of dependence before and after the event (the outbreak of the COVID-19 pandemic) in relation to mutual information (delta) and linear correlation, we saw the biggest differences for the German market. For the DAX sectors, linear correlation underestimates post-event dependencies. The dependencies for other countries were similar on average. For half of the sectors (all markets), we recorded an increase in dependence after the event. A sector where we recorded growth in all countries was the TECH sector.

Implications & Recommendations: The dependence measurement using mutual information expressed in terms of copulas has many advantages. It is not limited to measuring linear correlations. It can also capture a nonlinear correlation. Furthermore, it not only measures the dependence degree, but also considers the dependence structure, which is more than a correlation. Moreover, there was no assumption about the ellipticity of marginal and joint distribution. This dependence measure even allows for the modelling of the dependence of variables with different cumulative distribution functions.

Contribution & Value Added: The novelty of this article is that it compares the results of dependence measurements by linear correlations and mutual information expressed in terms of copula entropy. Considering the indices and subindices of the main European stock markets, when both measures of dependence were used, we obtained significantly different results in both subperiods under investigation (i.e. before and after March 11, 2020).

Article type:	research article
Keywords:	foreign main subindices; pandemic Covid-19; mutual information; copula entropy; linear correlation
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INTRODUCTION

The world has faced many medical crises, for example, SARS-COV in 2003, MERS-COV in 2012, and Ebola in 2014. However, the one that had the greatest influence on the world economy was COVID-19. The first reports on this disease were published in December 2019 in China.

China was the first country where COVID-19 broke out. China was also the first country in the world to implement measures to overcome the COVID-19 pandemic. Considering scientific investment methods, investors try to obtain higher profits and/or reduce investment losses. One of the best-known investment strategies is diversification. According to this strategy, assets are distributed to stocks from different sectors. Its main goal is to avoid investment losses caused by investing in closely dependent assets.

The COVID-19 pandemic was the source of the greatest recession in the world's economy since the global financial crisis of 2008. A very useful piece of advice for investors trying to optimise their investment during the COVID-19 pandemic was to identify the structure and changes in the interdependence between the various sectors.

The World Health Organization (WHO) announced the outbreak of the COVID-19 pandemic on 11 March 2020. Consequently, most countries in the world used various measures to slow down the spread of the virus. These measures significantly impacted many aspects of the behaviour and life of societies around the world. COVID-19 impacted global financial markets severely. The main observation was a slowdown in the global economy.

LITERATURE REVIEW AND HYPOTHESES DEVELOPMENT

Economic and social issues, which are related to one another, are the subject of numerous academic articles. In these contributions, the negative influence of the COVID-19 pandemic (Goodell, 2020 among others) especially on trade, tourism, transportation and employment has been proven (Leduc & Liu, 2020). Some compare the effects of the spread of COVID-19 and its consequences to those of an economic crisis (Sharif *et al*., 2020).

Some authors have proven the effects of the pandemic on the returns of financial markets (Ashraf, 2020; Zhang *et al*., 2020; Aslam *et al*., 2020 a,b,c) and/or their volatility (Albulescu, 2020; Bakas & Triantafyllou, 2020; Zaremba *et al*., 2020; Okorie & Lin, 2020).

Rizwan *et al*. (2020) proved the systemic risk to banking in eight leading economies. All of them were strongly impacted by COVID-19. The authors found that the systemic financial risk of these countries rose significantly during the pandemic period.

Some articles have focused on the performance of stocks in numerous sectors or countries. Mazur *et al*. (2020) investigated the return rate of the healthcare, food, natural gas, and software sectors. They found that the performance of these sectors was satisfactory during COVID-19. However, they detected that some sectors, such as the crude petroleum, real estate, entertainment and hospitality sectors, declined considerably. Moreover, these sectors exhibited great volatility.

Shehzad *et al*. (2020) compared the pandemic's effects on the stock market to the impact of global financial crises. They found that COVID-19 affected the American and European stock markets more strongly than the financial crisis. Moreover, the pandemic affected economic communication throughout the world and was the source of a financial crisis.

For market participants, it is crucial to analyse the interrelations in the stock market. This is significant concerning diversifying investment and building investment portfolios during the pandemic. It is important from the point of view of risk management of the financial market, which is considered by financial regulators.

In recent years, some authors have attempted to investigate the interdependence between stock markets (Sukcharoen & Leatham, 2016; Long *et al*., 2016; Qiao *et al*., 2016; Long *et al*., 2017a, b; Surya *et al*., 2018; Alomari *et al*., 2018; Ji *et al*., 2019; Kodres & Pritsker, 2002; Barberis *et al*., 2005; Chiang & Zheng, 2010; Wang & Hui, 2018). These researchers have applied, among other models, the GARCH model, the Copula model, Granger causality test, and the DCC model. The goal was to detect the interdependence structure between different stock sectors in the countries under consideration.

Contributions to the interrelation structure of the stock markets have detected which sector plays the most important role in a national economy in a country under investigation. These studies provide new opportunities for investors to build an optimal assets portfolio (Poynter *et al*., 2015). Moreover, in Europe, there are few studies concerned with the interdependence structure of stock sectors during the period of the COVID-19 pandemic.

The copula entropy applied in this article consists of copula theory and information theory. We can summarise the axiomatic properties of copula entropy in the following points: they are multivariate, symmetric, non-negative, they display zero if and only if independent, they are invariant to monotonic transformation and equivalent to the correlation coefficient in Gaussian cases. The advantages of using copula entropy are the following: they are model-free, distribution-free, non-parametric, tuning-free, insensitive to parameters, they converge well, they are easy to implement, there is a low computation burden, they are interpretable with physical meanings and are supported by rigorous mathematics. Copula entropy can measure association information and dependence structure information simultaneously. The copula function describes the dependence between variables. Mutual information is applied to quantify the dependence. There is a relation between copula theory and information theory. We can express mutual information as copula entropy, *i.e.* in terms of copulas. Copula entropy does not impose restrictions on the dimension of multiple variables.

One of the first and best-known contributions using copula entropy is Zhao and Lin's article (2011). In their article, they constructed the copula entropy model based on the copula and entropy theory. Thus, the copula entropy model reflected the advantages of both of them. Their method is useful in measuring not only the linear correlation, but also the nonlinear one. Furthermore, it measures not only the dependence degree, but also its structure. Zhao and Lin suggest copula entropy models with two and three variables. The goal was to measure dependence on stock markets. This approach is an extension of copula theory and is based on Jaynes's information criterion. The research sample consisted of 12 stock indices from 12 countries selected using two methods. Zhao and Lin selected respective copula functions to represent three different economic situations: recession, boom, and interim. Having completed the two experiments, they provided a comparative analysis. The authors established that changes in three-variable dependence across the three economic situations are less obvious than in the case of two-variable dependence. Zhao and Lin (2011) used the copula entropy model to measure stock market correlations, compared with the linear correlation coefficient and mutual information methods, which have the advantages of being dimensionless and can capture non-linear correlations.

Ma and Sun (2011) proved the equivalence between copula entropy and mutual information. They showed that mutual information is essentially an entropy. This article suggests a new way of understanding and estimating mutual information using the copula function. The authors define the entropy of the copula, as called copula entropy. This is defined as a measure of the dependence uncertainty represented by the copula function. Then mutual information is shown to be equivalent to negative copula entropy. With this equivalence, mutual information can be estimated – as the authors demonstrate – by first estimating the empirical copula and then estimating the entropy of the empirical copula. Therefore, the mutual information estimate is an estimation of the entropy, which reduces the complexity and computational requirements. Tests demonstrate that this method is more effective than the traditional one.

This article concerns the dependence of the US, British, German and French subsectors of the stock markets during the pandemic period. Our goal – to study the subsectors of the leading economies around the time of the outbreak of COVID-19 on 11 March 2020 – seems to be interesting. Moreover, the results may be useful for investors operating in these markets.

Our main research question concerned how the dependencies of subindices changed before and after the event day (11 March 2020 – on that day the World Health Organisation declared the state of epidemic threat throughout the world). We investigated the dependence of the subindices of world-leading stock markets using the approach of mutual information before and after the event day. Based on the literature review, we formulated the following hypothesis:

H1: The dependencies of the subindices of world-leading stock markets were essentially greater after the event day.

We investigated whether dependence behaviour at the time was similar for the indices and subindices of the countries under consideration. For this goal, we used mutual information based on copula entropy. Another research problem concerned changes over time in Pearson's correlation and mutual information concerning event day. This leads us to the second hypothesis:

H2: The use of mutual information measure and linear correlation produced quite different results in the period under study.

To verify these hypotheses, we compared the results of mutual information and Pearson's correlation before and after event day and explained the possible differences concerning linear and nonlinear dependence notions.

After Introduction, Literature Review and Hypotheses Development we start with the presentation of Research Methodology. Based on the described methodology we conduct empirical analysis and discuss the results in the chapter Results and Discussion. In the final part of the paper we summarised main results and indicate further studies.

RESEARCH METHODOLOGY

Mutual Information and Copula Entropy

One of the most important terms of information theory is entropy, which measures the average amount of information and mutual information that is used to model dependence. For discrete random variable X entropy is given by:

$$H(X) = -\sum_{x \in \mathcal{X}} p(x) \log_m p(x) \tag{1}$$

in which $\mathcal{X}$ is the support set and $p(x)$ is the probability of the event x. The unit of entropy is bit if the logarithm base is equal to 2 (if the base is e then the unit is called nat and if 10 then the unit is dit). Conditional entropy measures the entropy of variable Y when the values of X are known, and is given by:

$$H(Y|X) = -\sum_{y \in \mathcal{Y}} \sum_{x \in \mathcal{X}} p(x,y) \log \frac{p(x,y)}{p(x)} \tag{2}$$

in which $p(x,y)$ and $p(y)$ represent joint and marginal probability. Given the above definitions, mutual information is given by $MI(X,Y) = H(X) - H(Y|X)$, which in terms of distributions and joint probability distribution is given by:

$$MI(X,Y) = \sum_{y \in \mathcal{Y}} \sum_{x \in \mathcal{X}} p(x,y) \log \frac{p(x,y)}{p(x)p(y)} \tag{3}$$

Mutual information measures the amount of information of one variable contained in another and is non-negative and bounded by entropies of each variable but not normalised (like the Pearson correlation coefficient). Following Joe (1989), to normalise mutual information one can use transformations using the formula $\delta = \sqrt{1 - \exp(-2MI)}$ (Joe, 1989). The definitions given above are reformulated in terms of integrals for continuous variables.

The term copula entropy combines both information and copula theories (for definition and properties of copulas see for example Nelsen, 2006). For bivariate copula with density $c(u,v)$ is given by:

$$H_C(U,V) = -\int_0^1 \int_0^1 c(u,v) \log c(u,v)\, du dv \tag{4}$$

Ma and Sun (2011) proved that copula entropy is equal to the negative of mutual information, $MI = -H_C$. From the equation above, we see that copula entropy is the expected value of the logarithm of copula density and the double integral can be computed as a mean:

$$H_C(U,V) = -E[\log c(u,v)] = -\frac{1}{n} \sum_{t=1}^{n} \log c(u_t, v_t) \tag{5}$$

Ma and Sun (2011) also implemented a method for estimating mutual information in a non-parametric way. It is worth mentioning that for some families of copulas mutual information is expressed explicitly. An example is the Gaussian copula for which mutual information is equal to $-\frac{1}{2}\ln(1-\rho^2)$.

The Database and Descriptive Statistics

We consider the closing prices of the sectoral indices for France (CAC sectors), Germany (DAX sectors), the UK (FTSE sectors) and the US (SP sectors), along with the main indices from these countries, that is CAC40, DAX, SP500, and FTSE100 (we collected the data from investing.com database). Tables 1, 2, 3, and 4 provide sector abbreviations.

Table 1. Full names and abbreviations of CAC sectors

Sector number	Name of CAC sector	Abbreviation
	Basic Materials	BASIC
2	Consumer Goods	CGOOD
3	Consumer Service	CSERV
4	Financials	FIN
5	Health Care	HEALTH
6	Industrials	IND
7	Oil & Gas	OIL
8	Technology	TECH
9	Utilities	UTIL

Source: investing.com

Table 2. Full names and abbreviations of DAX sectors

Sector number	Name of DAX sector	Abbreviation
1	Automobile	AUT
2	Banks	BANK
3	Chemicals	CHEM
4	Consumer&Cyclical	CONS
5	Financial Services	FIN
6	Industrial	IND
7	Insurance	INS
8	Media	MED
9	Pharmaceuticals&Healthcare	PHA
10	Retail	RET
11	Software	SOFT
12	Technology	TECH
13	Telecom	TELE
14	Transportation&Logistics	TRAN
15	Utilities	UTIL

Source: yahoo.finance.com

Table 3. Full names and abbreviations of FTSE sectors

Sector number	Name of FTSE sector	Abbreviation
1	Banks	BANK
2	Basic Resources	BAS
3	Chemicals	CHEM
4	Cons.& Materials	CONST
5	Food & Beverage	FOOD
6	Health Care	HEALTH
7	Ind. Goods & Ser.	INDGS
8	Insurance	INS
9	Media	MED
10	Oil & Gas	OIL
11	Per. & Household	PERS
12	Retail	RET
13	Technology	TECH
14	Telecom.	TELE
15	Travel & Leisure	TRAV
16	Utilities	UTIL

Source: investing.com.

Table 4. Full names and abbreviations of SP sectors

Sector number	Name of SP sector	Abbreviation
1	Communication Services	COMM
2	Consumer Discretionary	CONSD
3	Consumer Staples	CONSS
4	Energy	ENERG
5	Financials	FIN
6	Health Care	HEALTH
7	Industrials	IND
8	Information Technology	INF
9	Materials	MAT
10	Real Estate	REST
11	Utilities	UTIL

Source: yahoo.finance.com

On 11 March 2020, the World Health Organisation (WHO) announced the COVID-19 pandemic. We divide the time series of logarithmic returns with this date and refer to this as event day. For all series, we computed descriptive statistics (mean, standard deviation, kurtosis, and skewness) along with normality and autocorrelation testing. The Jarque-Bera tests confirmed departure from normality.

RESULTS AND DISCUSSION

Main Index: Sectoral Index Dependence

We investigate the strength of dependence by computing mutual information for all sectoral indices and the main index for a given country. We computed this measure using copula entropy. We divided the time series of returns with the event day and for each calculated the probability integral transform. To do this, we filtered our time series using vector autoregression models for conditional means, GARCH type models for conditional variance and skew *t* for conditional distributions. We selected the copulas that fit the best from the selected families. We limited the choice of copulas to a set investigated by Tenzer and Elidan (2016). These authors established a monotonic relationship between the mutual information and the copula dependence parameter. This means that the strength of dependence increases as the parameter increases. We limited the set of potential copulas to those investigated by the authors but in the case of Archimedean copulas we added their rotated versions (survival copulas). We chose copulas that fit the best dataset using the Bayesian information criterion. For comparison purposes, for all pairs, we computed the linear correlation coefficient (for raw returns). In Figure 1, we present the strength of dependence according to the delta parameter and linear correlation coefficient (red before the event, green after the event).

Of the CAC sectors UTIL (0.54) and IND (0.91) had, respectively, the lowest and highest values before the event, whereas after the event HEALTH (0.57) and IND (0.91) had the lowest and highest values. In six cases, dependence was greater after the event with the largest percentage change (41%) in the case of UTIL. We observed the highest drop for HEALTH (about 19%). When we consider linear correlation, the situation is identical with some different values. For DAX, the results based on delta and linear correlation were different. Whereas the sectors with the weakest (RET) and strongest (CHEM) dependence after the event were the same (0.32 and 0.91 for delta and 0.26 and 0.93 for linear correlation), before the event, we noticed some difference. The minimal values of delta and correlation were both for the UTIL sector (0.44 and 0.71) but the maximum was for CHEM (0.88) and INS (0.93). We noted the main difference when considering dependence changes in terms of percentage. In 10 of 15 cases, the delta parameter was greater after the event (with the highest value of change 43% for UTIL and the lowest one for RET -51%). For the linear correlation, the lowest was also for RET (-64%) but the highest was for CONS with only 3%. Only in three cases, dependence was greater after the event. For FTSE sectors, INDGS had the highest delta value either before or after the event with values of 0.86 and 0.88 respectively (the lowest ones were UTIL 0.47 and TECH 0.57). Linear correlation coefficients give similar indications: the highest value before and after for INDGS (0.9 and 0.91), and for INDGS and HEALTH the lowest (UTIL with a value 0.56 and HEALTH with a value 0.62). The vast majority of sectors have positive percentage changes with the highest ones for FOOD and UTIL for delta (32% and 25%) and UTIL and FOOD for correlation (25 and 21). In the case of SP sectors, we noticed the same sectors with minimal and maximal values of dependence parameters either before (UTIL and INF, respectively) and after the event (ENRG and INF). For the delta parameter, these values before the event were 0.31 and 0.91, whereas for linear correlation – 0.62 and 0.95. After the event, they were 0.59 and 0.89 for delta and 0.72 and 0.94 for correlation. Both for delta and correlation, we noticed an increase in dependence for the six sectors with the highest values for UTIL (102% for delta and 30% for correlation). We observed the largest decrease in dependence for ENERG (about 9% for both measures of dependence).

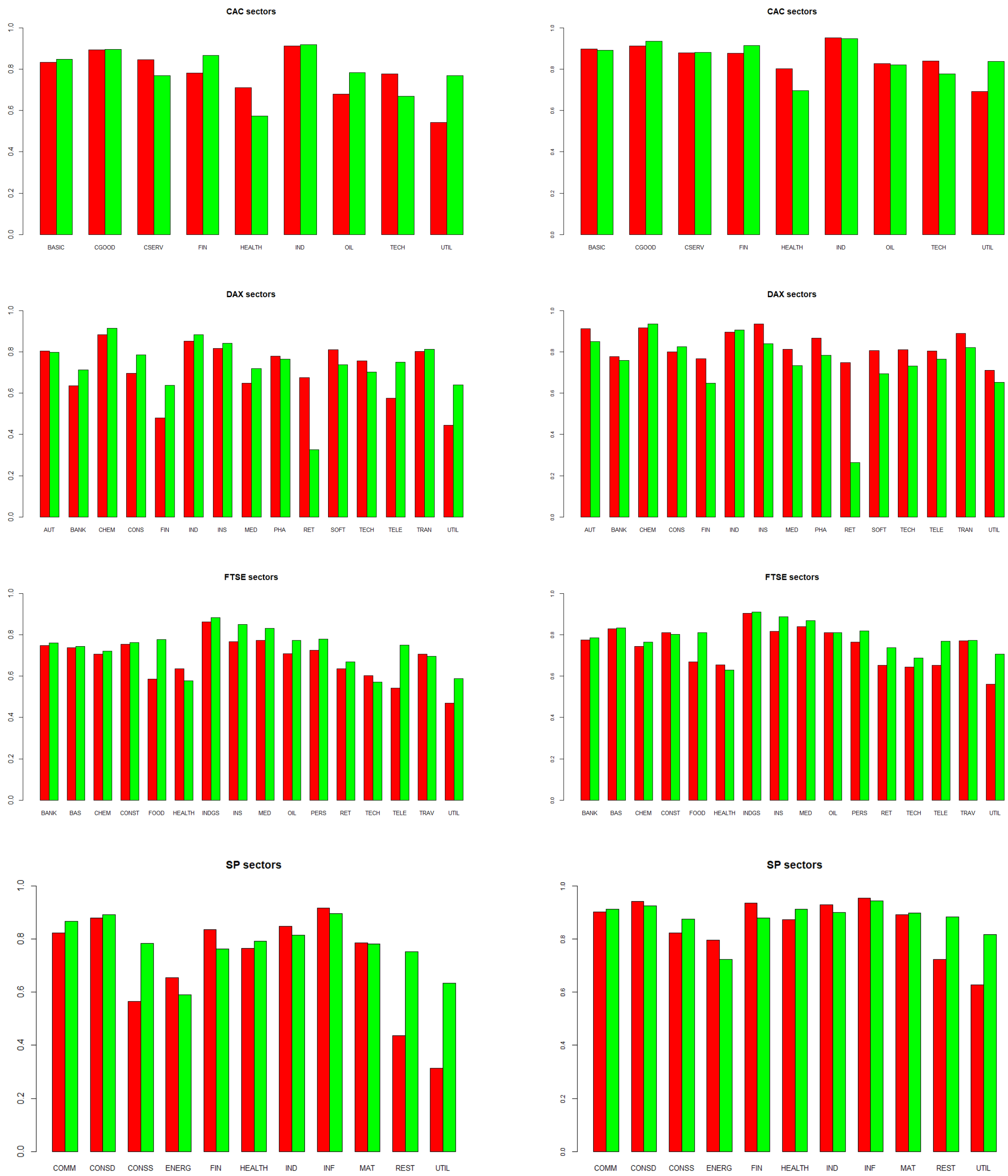

Figure 1. Delta (left) and linear correlation (right)
Source: own elaboration.

Dependence of Subindices

In this section, we investigated the dependence between subindices for a given country. We applied the methods described in the previous sections. Due to the large number of pairs, we present the degree of dependency in heatmaps. Figure 2 presents computed delta parameters (before and after the event).

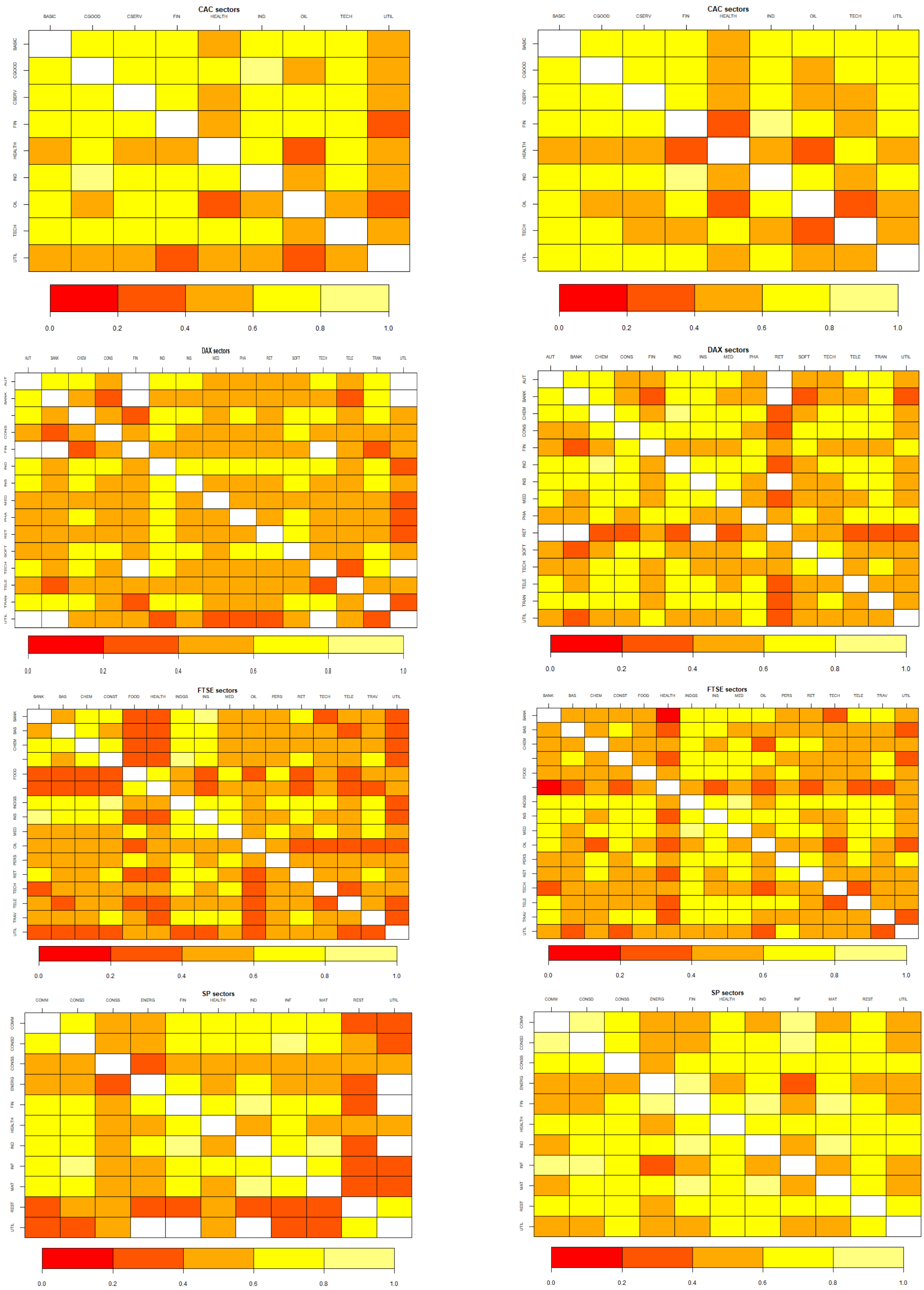

Figure 2. Heat maps of parameters delta before (left) and after (right) the event (from top to bottom France, Germany, GB and USA)

Source: own elaboration.

After the event, there is a general increase in dependence between sectors for DAX, FTSE, and SP. The number of pairs with greater dependence was 63 pairs out of 105 for DAX, for FTSE 75 out of 128, and for SP 38 out of 55.

In Table 5, we present the three weakest (bottom rows) and three strongest (top rows) relationships before and after the event.

Table 5. Selected weakest and strongest relationships based on the delta parameter

Before event		After event	
CAC sectors			
IND - CGOOD	0.81	IND - FIN	0.85
TECH - IND	0.79	OIL - FIN	0.79
CSERV - CGOOD	0.78	CGOOD - BASIC	0.76
OIL - HEALTH	0.40	TECH - OIL	0.38
UTIL - FIN	0.39	HEALTH - FIN	0.35
UTIL - OIL	0.38	OIL - HEALTH	0.32
DAX sectors			
IND - CHEM	0.77	IND - CHEM	0.82
TRAN - AUT	0.74	INS - IND	0.80
TRAN - IND	0.73	IND - BANK	0.79
TECH - FIN	0.28	RET - CHEM	0.26
UTIL - AUT	0.27	TELE - RET	0.22
UTIL - TECH	0.26	RET - IND	0.21
FTSE sectors			
INS - BANK	0.82	MED - INDGS	0.85
INDGS - CONST	0.81	INS - INDGS	0.78
INS - CONST	0.76	INDGS - CONST	0.77
RET - HEALTH	0.24	UTIL - TRAV	0.24
UTIL - OIL	0.22	TRAV - HEALTH	0.23
UTIL - CHEM	0.21	HEALTH - BANK	0.18
SP sectors			
MAT - IND	0.81	IND - FIN	0.89
INF - CONSD	0.80	MAT - IND	0.89
IND - FIN	0.80	INF - CONSD	0.87
UTIL - CONSD	0.27	ENERG - COMM	0.43
UTIL - INF	0.26	UTIL - ENERG	0.42
REST - ENERG	0.25	INF - ENERG	0.37

Source: own study.

The subindices that form pairs for the strongest and weakest relationships were largely the same before and after the event. The strongest relationships were for SP sectors and this was the only case when the weakest relationship after the event was stronger than the weakest one before the event. Among the sectors for which we noted the weakest dependencies before the event, there was often UTIL, while among the strongest after the event there was IND.

It is interesting to see how much dependency increases. In Figure 3 below, the percentage changes were presented as heatmaps. Table 6 gives the three strongest and weakest changes.

We noticed that the largest percentage changes were the smallest in general for the CAC sectors and the largest for the SP sectors. We observed the smallest changes again for the CAC sectors, but the largest ones for the DAX sectors. Given δ_{ij}, the dependence parameter between i and j subindices, we computed $S_i = \sum_j \delta_{ij}$, which reflects the sum of parameters δ of a certain subindex with all of the other subindices (presented in Figure 4).

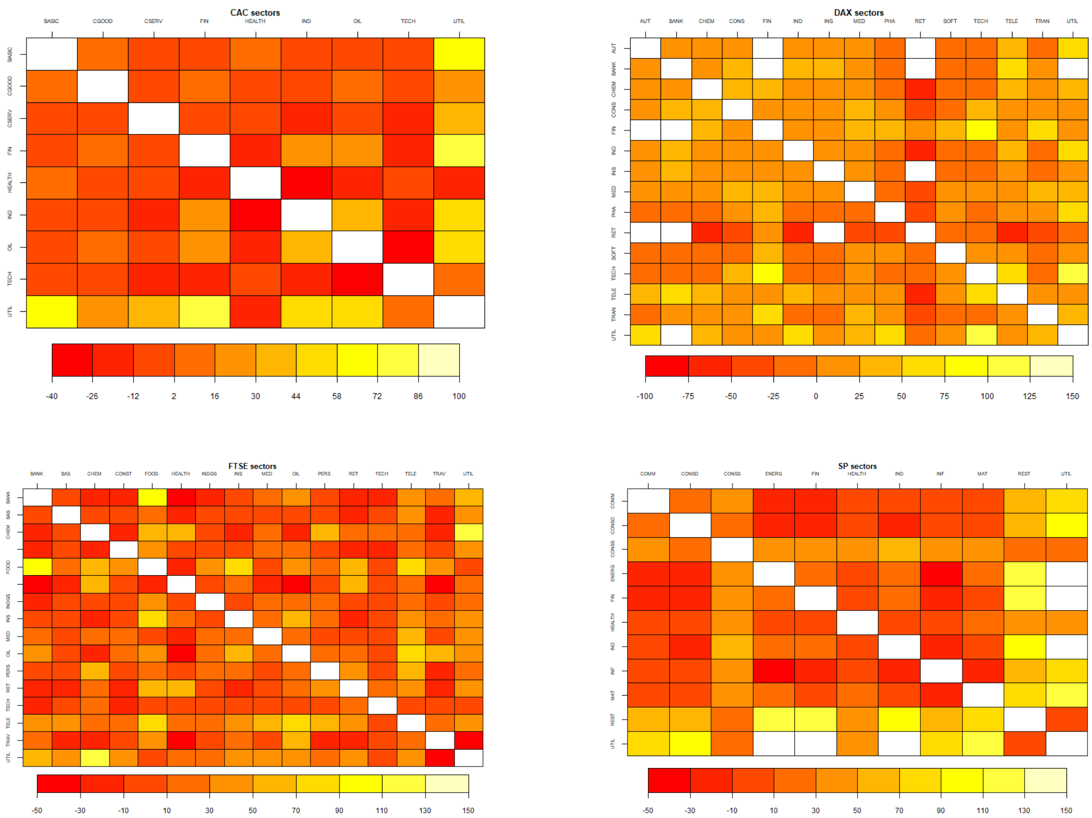

Figure 3. Heat maps of percentage changes of parameter δ
Source: own elaboration.

Table 6. Selected smallest and largest percentage changes of δ

CAC sectors		DAX sectors	
UTIL - FIN	80.96	UTIL - TECH	103.01
UTIL - BASIC	64.24	TECH - FIN	99.8
UTIL - OIL	55.01	TELE - TECH	64.91
TECH - IND	-25.17	TELE - RET	-50.42
HEALTH - FIN	-25.77	RET - CHEM	-56.45
TECH - OIL	-29.52	RET - IND	-67.3
FTSE sectors		**SP sectors**	
UTIL - CHEM	129.85	REST - FIN	125.54
FOOD - BANK	91.81	UTIL - MAT	118.34
TELE - OIL	79.79	REST - ENERG	114.92
UTIL - TRAV	-39.18	FIN - CONSD	-22.04
TRAV - HEALTH	-40.5	INF - FIN	-28.68
HEALTH - BANK	-44.53	INF - ENERG	-32.15

Source: own study.

We can see from Figure 4 above that for five out of nine CAC sectors S_i was greater after the event (BASIC, CGOODS, FIN, OIL and UTIL) with the greatest percentage change for UTIL (almost 35%). The smallest percentage change was for the HEALTH and TECH sectors (about 13%).

For DAX sectors, only S_i was. smaller after the event only in three cases (INS, SOFT and RET with the largest decrease 48%). We noted the largest increase of more than 50% for FIN and UTIL. In the case of FTSE sectors in only 4 out of 16 (BAS, HEALTH, TECH, TRAV) the percentage change was negative (for HEALTH it is about 3%). The FOOD and TELE sectors increased the most (about 30%). In the case of SP sectors, the situation was different and we noticed a positive change for all sectors with the highest value over 100% for UTIL (the smallest for INF with about 2%).

For purposes of comparison, we computed the linear correlation coefficients for raw stock returns. We present illustrations of these calculations with similar figures and tables. The heat map in the Figure 5 below illustrates the strength of dependence between all sectors according to linear correlation. We based the legend for these figures on the minimum and maximum of all dependencies.

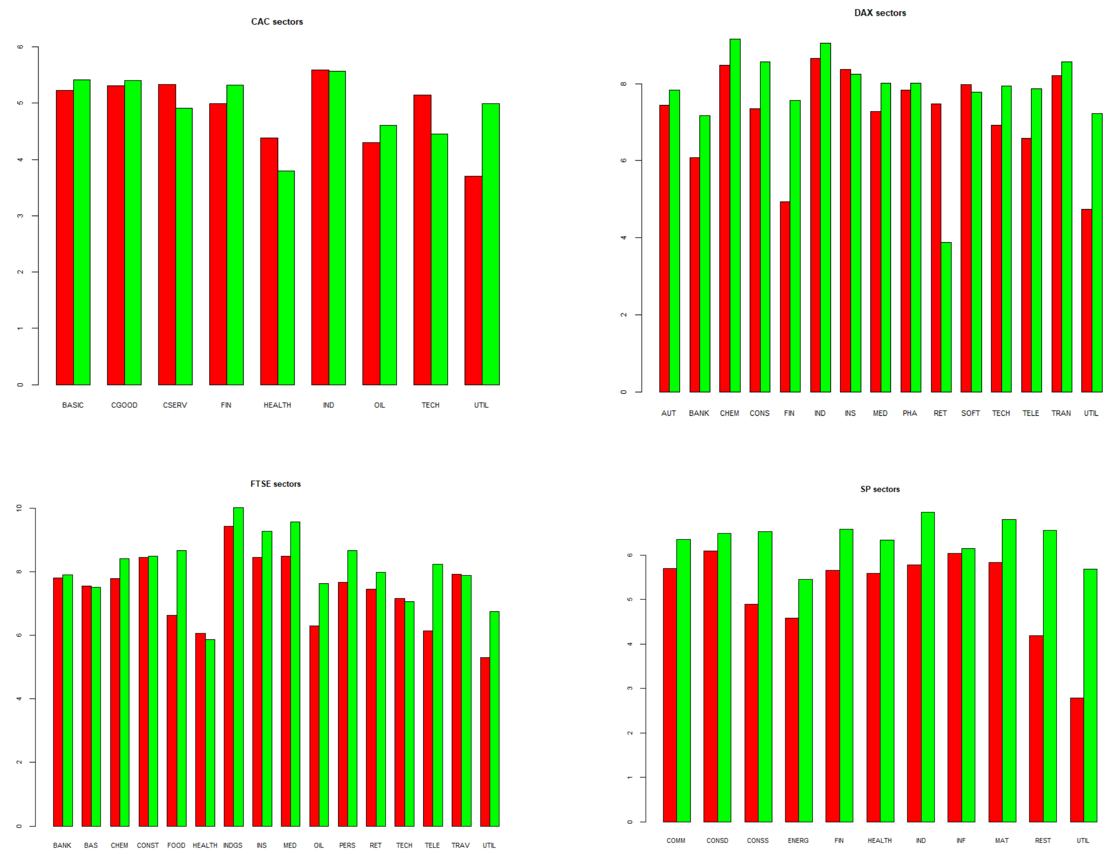

Figure 4. S_i of each sector before (red) and after the event (green)
Source: own elaboration.

In the case of CAC sectors, the number of pairs for which correlations after the event increases was equal to 16 and in all cases correlation coefficients are positive. For DAX sectors, dependence was greater only for 8 pairs after the event and for one pair (RET – BANK), we observed a negative but small correlation. Both for FTSE and SP, the number of pairs for which we observe an increase in dependence exceeded 70%. Furthermore, pairs of subindices for which we noted the strongest and weakest relationships were similar to those from mutual information, with the exception of the weakest relations for DAX sectors (presented in Table 7).

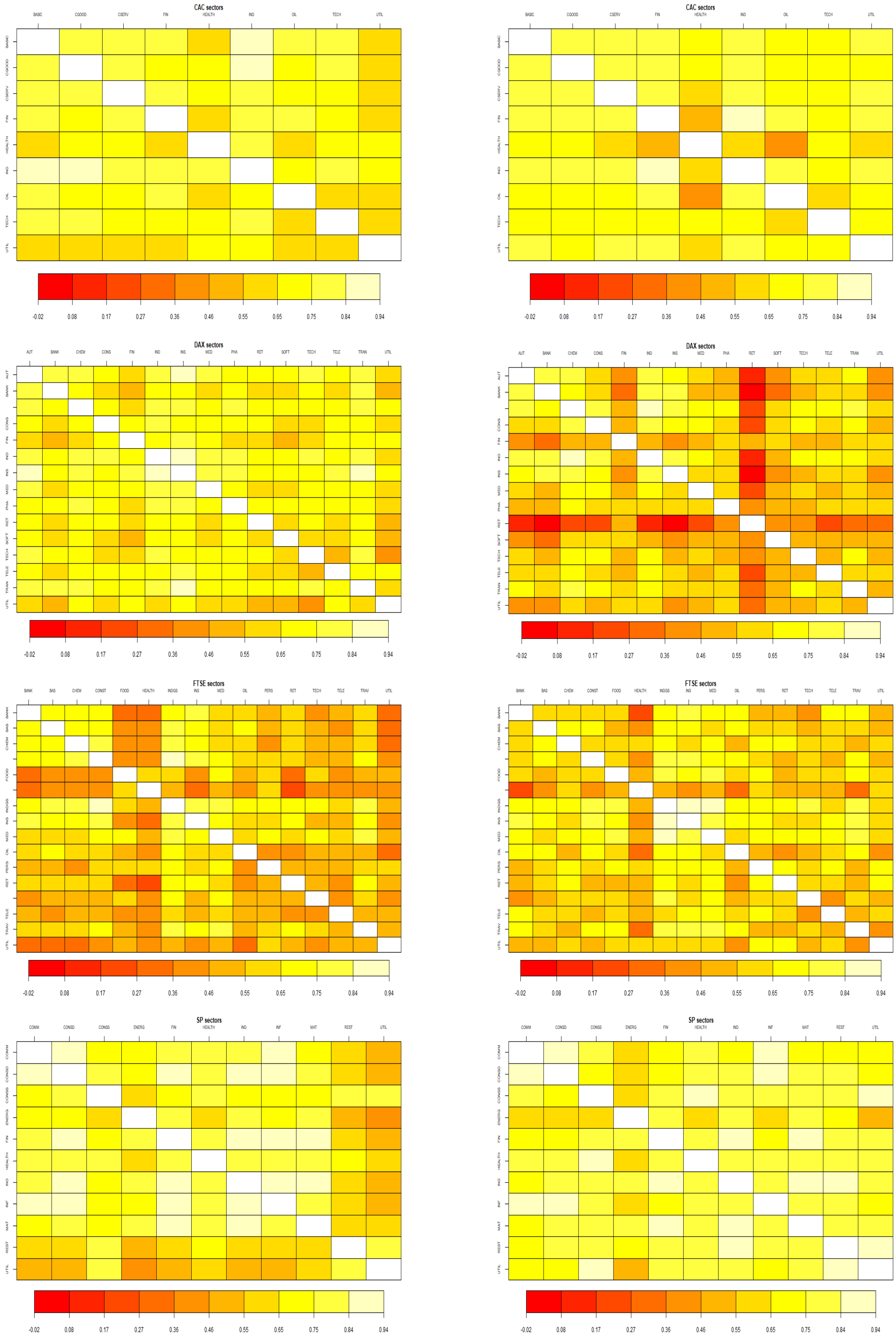

Figure 5. Heat maps of correlation coefficients before (left) and after (right) the event

Source: own elaboration.

Table 7. Selected weakest and strongest relationships based on linear correlation

Before event		After event	
CAC sectors			
IND - BASIC	0.85	IND - FIN	0.90
IND - CGOOD	0.85	IND - CGOOD	0.84
CSERV - CGOOD	0.84	CSERV - BASIC	0.83
UTIL - BASIC	0.56	TECH - OIL	0.56
UTIL - FIN	0.56	HEALTH - FIN	0.48
UTIL - OIL	0.55	OIL - HEALTH	0.42
DAX sectors			
INS - AUT	0.87	IND - CHEM	0.85
INS - IND	0.86	IND - AUT	0.81
TRAN - INS	0.85	INS - IND	0.81
UTIL - RET	0.51	RET - AUT	0.10
FIN - BANK	0.50	RET - INS	0.01
UTIL - TECH	0.45	RET - BANK	-0.02
FTSE sectors			
INDGS - CONST	0.85	INS - INDGS	0.87
INS - BANK	0.84	MED - INDGS	0.86
MED - INDGS	0.83	TRAV - INDGS	0.82
INS - HEALTH	0.29	OIL - HEALTH	0.35
UTIL - CHEM	0.27	TRAV - HEALTH	0.30
RET - HEALTH	0.22	HEALTH - BANK	0.26
SP sectors			
IND - FIN	0.91	IND - FIN	0.93
INF - CONSD	0.91	MAT - IND	0.93
MAT - IND	0.89	INF - CONSD	0.91
UTIL - INF	0.51	ENERG - CONSS	0.56
UTIL - COMM	0.51	INF - ENERG	0.56
UTIL - ENERG	0.44	UTIL - ENERG	0.55

Source: own study.

As above, we computed percentage changes in dependence (now based on the linear correlation coefficient). Figure 6 and Table 8 below show the results (the smallest and largest changes).

The scale of changes was similar only for the highest values of FTSE changes and for the smallest changes in the CAC and SP sectors. Similarly to S_i, we computed the sum of correlation coefficients between a given sector and all the other sectors. Figure 7 presents the results.

In the case of CAC sectors, we noted the same cases for which the sum of the correlation coefficients increased, with the highest increase for UTIL (about 22%). The smallest value was for HEALTH (about 10%). For all DAX sectors, the sum of the coefficients was smaller after the event, with the highest percentage change for RET (about 61%), which we can clearly see in the figure above. In the case of FTSE sectors, all percentage cases were positive with about a 30% increase for FOOD and UTIL (the smallest increase is noted for TRAV, with a value of 1.5%). For two SP sectors (CONSD and ENERG), the change was negative but not greater than 1% (the smallest for INF with about 2% and the highest for UTIL with a 27% change).

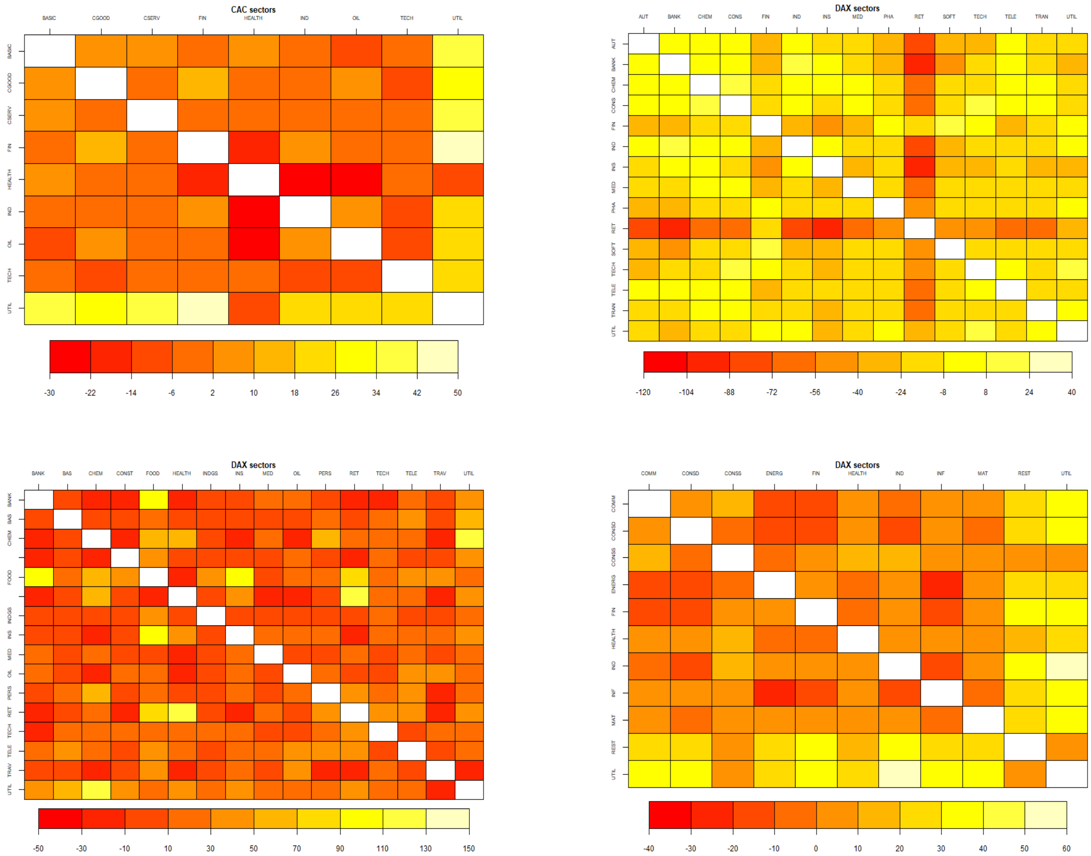

Figure 6. Heat map of percentage changes of correlation coefficients
Source: own elaboration.

Table 8. Selected smallest and largest percentage changes in correlation coefficient

CAC sectors		DAX sectors	
UTIL - FIN	45	IND - BANK	21.18
UTIL - BASIC	35.29	UTIL - TECH	16.03
UTIL - CSERV	35.26	CONS - CHEM	15.39
HEALTH - FIN	-18.74	RET - AUT	-84.02
IND - HEALTH	-23.46	RET - INS	-98.11
OIL - HEALTH	-24.53	RET - BANK	-103.29
FTSE sectors		**SP sectors**	
RET - HEALTH	117.58	UTIL - IND	52.45
UTIL - CHEM	113.35	UTIL - MAT	39.4
INS - FOOD	100.25	UTIL - FIN	38.04
TRAV - RET	-21.28	INF - FIN	-15.7
CONST - CHEM	-22.18	ENERG - CONSD	-16.11
TRAV - HEALTH	-26.62	INF - ENERG	-20.99

Source: own study.

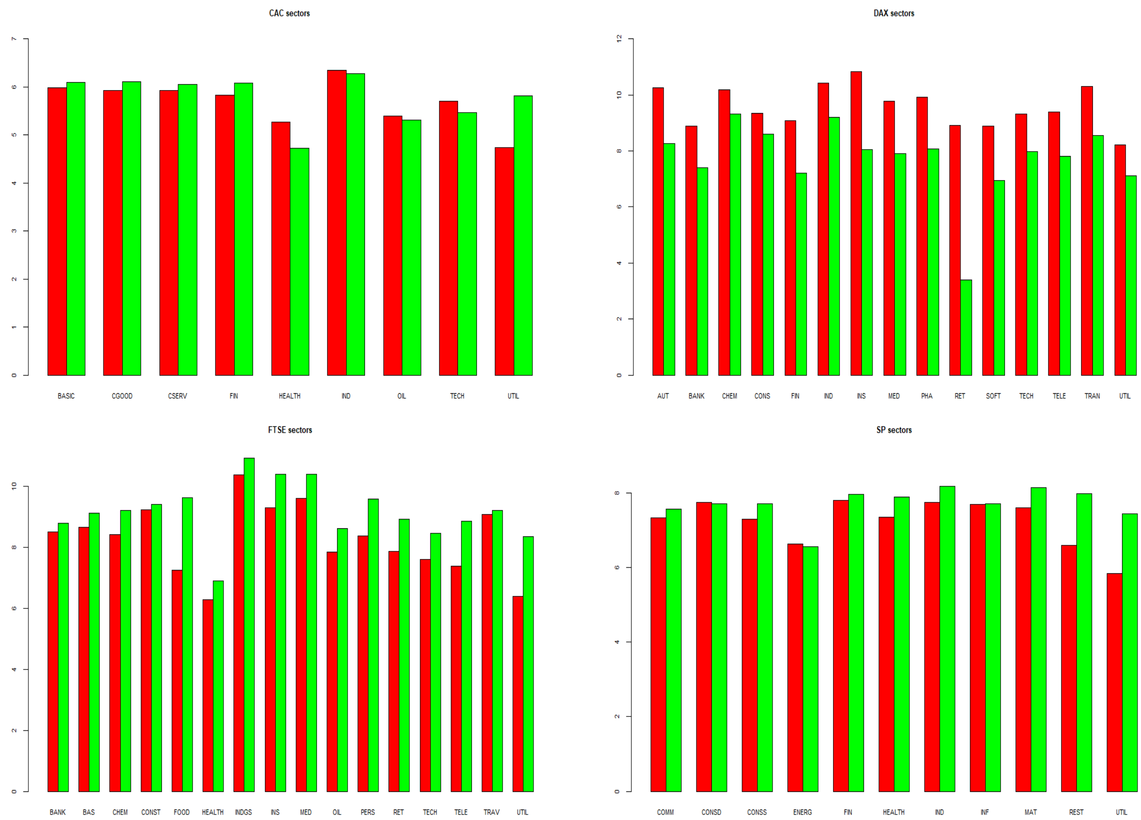

Figure 7. Sum of correlation coefficients of each sector before and after the event
Source: own elaboration.

CONCLUSIONS

To summarise, dependence measurement using mutual information expressed in terms of copulas has many advantages. It is not limited to measuring linear correlations. It can also capture a non-linear correlation. It measures the degree of the dependence and considers the dependence structure, which is more than correlation. Moreover, there is no assumption about the ellipticity of marginal and joint distribution. It even allows the dependence of variables with different cumulative distribution functions to be modelled.

When it comes to indications of the strength of dependence before and after the event in relation to mutual information (delta) and linear correlation, we saw the biggest differences for the German market. For DAX sectors, linear correlation underestimated post-event dependencies. The dependencies for other countries were similar on average, for half of the sectors (all markets), we recorded an increase in dependence after the event. In all countries, we recorded growth in the TECH sector.

The subindices that form pairs for the strongest and weakest relationships were largely the same before and after the event both for mutual information and linear correlation. The strongest relationships were SP sectors, and this is only the case when the weakest relationship after the event was stronger than the weakest one before the event. We noted the weakest dependencies before the event with pairs with UTIL, while the strongest ones after the event – with IND.

When considering positive percentage changes based on measuring mutual information, we noticed that the largest ones were for the SP sectors and the smallest ones for the CAC sectors. We observed the smallest changes for the CAC sectors but the largest ones for DAX. For linear correlation, the scale of percentage changes was similar only for the highest values of FTSE changes and for the smallest changes in the CAC and SP sectors.

For the sum of the dependence parameters of a subindex with all of the other subindices, we observed the clearest situation for SP sectors, for which we noticed a positive change for all sectors with the highest UTIL (for CAC and DAX this is also the case). For correlation, the similar parameter was smaller after the event for all DAX sectors and this was the biggest difference with respect to mutual information. Interestingly, the UTIL sector recorded the largest positive changes, as was the case with the measure based on mutual information.

In most cases, financial time series had a dependence structure that could not be captured by models based on elliptical distributions. Another problem came from the dynamic behaviour of conditional moments of the time series. For this reason, to properly describe the dependence structure we recommend using copula and information theories.

In this article, we use static copulas. However, the parameters that reflect dependencies can be dynamic over time. Another limitation of this study was the lack of high-frequency data with respect to subindices.

In further studies, high-frequency data should be used to describe risk represented by conditional variances (realized variances) of time series and models based on both dynamic copulas and an information theory approach.

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